Dynamic Policymaking with Decay*

Steven Callander† Gregory J. Martin‡

September 3, 2015

Abstract

It is often said that the only constant is change itself. As time passes, the population grows, new technologies are invented, and the skills, demographics, and norms of the populace evolve. These changes, whether in isolation or in aggregate, impact the effectiveness of policy. In particular, policies designed for today’s world are unlikely to provide a perfect fit tomorrow. In this paper we develop a notion of policy decay that captures this impact formally. We introduce policy decay into a simple, paradigmatic model of legislative policymaking and show that it leads to a starkly different perspective on legislative politics. Our results upend the classic notion of gridlock, and bear implications more broadly for the practice of politics. We show in turn how a changing world impacts the power of agenda control, how it drives the dynamic path of legislation, how it reveals a novel yet empirically relevant notion of expertise, and how it, ultimately, provides a foundational logic to the design of bureaucracy.

*An early version of this paper was presented at the Conference on Executive Politics at Washington University in St. Louis, June 2014. We thank our discussant, Nolan McCarty, as well as Alan Wiseman, Terry Moe, Will Howell, and other members of the audience for helpful comments. We also benefitted from comments in seminars at Emory University, Stanford University, and the meetings of the Midwest Political Science Association and Southern Political Science Association.
†Stanford Graduate School of Business. sjc@stanford.edu.
‡Emory University. gregory.martin@emory.edu.
1 Introduction

Political science is not a field of laws. Yet some theories rise to such levels of acceptance that they can be considered settled truths if not laws. One such theory is the theory of gridlock. In modern separation-of-powers systems, the logic goes, the multiplicity of veto points makes passing new laws at best difficult and at worst impossible. The theory has been developed nowhere more thoroughly than in the context of American politics and it has engaged the minds of the leading lights of the field (Mayhew (1991, 2005); Fiorina (1996); Krehbiel (1996); Brady and Volden (2006); etc.).

The theory of gridlock has also received the imprimatur of formal theorists. This has allowed the predictions of the theory to be made precise. In particular, it has led to the notion of a gridlock interval in the policy space, where the boundaries of the interval are given by the ideal points of the most extreme of the pivotal actors. For instance, in a reduced-form model of the Senate, the need to overcome a minority-led filibuster implies that the gridlock interval is the region between the ideal points of the 41st and 60th senators. Any policy that resides inside the gridlock interval is then immune from change. Moreover, policies that lie outside the gridlock interval face an irresistible pressure to be changed, with the change moving policy to within the gridlock interval, and the new policy untouchable thereafter. According to the logic of gridlock, therefore, the gridlock interval acts as an absorbing zone that exhibits a vice-like grip on policy. Each policy issue is inexorably dragged into the gridlock interval, never again to leave or be altered at all.

This logic is clear and compelling, and, within the confines of well-specified formal models, unassailable. But from the broader perspective of American politics, it does lead one to wonder what it is that the modern Congress does. If it is the case that a statute truly is immovable once it has entered the gridlock interval, why is it not the case that every law has already moved in to the gridlock interval? Since 1789 the U.S. Congress has enacted more than 20,000 statutes, yet it continues to pass laws at a regular rate of several hundred in each new Congress. To be sure, many of these new statutes are what Krehbiel (1998) calls “non-binding ‘hurrah’ resolutions,” yet many are substantive and non-trivial refinements of

\[\footnote{A more complete model of American politics requires, of course, that we add the committee system, majority party agenda power, the House of Representatives, as well as the President.}\]
existing statutes. If an area of the law is truly closed once it moves into the gridlock interval, why has the Congress not run out of things to legislate about?

We pose this question somewhat tongue-in-cheek, yet we do so with a serious intent as the question illuminates an issue with U.S. policymaking that is important and so far unexamined. If it is sensible to view Congress as processing an exhaustible list of issues, what is the force that supplies the constant, and apparently never ending, stream of legislative material? More importantly, how does this force (or forces) affect our understanding of the policymaking process and our predictions for policy outcomes? Exploring these questions, and expanding our conception of policymaking in a separation of powers system, are the objectives of the present paper.

In many respects the answer to our question is obvious: Change. One important type of change is the identity of the policymakers themselves. Elections bring change in the form of new presidents and new legislators and, in the aggregate, these changes can lead to movement of the gridlock interval, such that some policies that were inside are now outside the gridlock interval and subject to legislative action. Yet a closer examination indicates this is at best an incomplete story. Analysis of Congressional policymaking reveals no correlation between shifts in the gridlock interval and legislative activity. Moreover, movements in the gridlock interval following elections are typically minor. Even “paradigm-shifting” elections – such as those of 1932, 1980, and 2008 – are better described as bumps to the gridlock interval than earthquake-sized movements.\(^2\)

In this paper we examine a different kind of change: change in the world around us. As time passes, the population grows, new technologies are invented, and the skills, demographics, and norms of the populace evolve. These changes, whether in isolation or in aggregate, impact policy. In particular, policies designed for today’s world are unlikely to provide a perfect fit tomorrow. We capture this kind of impact formally through the notion of policy decay. We suppose that over time, inexorably and potentially stochastically, the fit of policy

\(^2\)For instance, across the 20th century, the gridlock interval never shifted so much that all policy positions were subject to change (Woon (2009)). The intersection of gridlock intervals is, therefore, non-empty and significant. Moreover, there is good theoretical reason to believe that most policies are centrally located in the gridlock interval (Romer and Rosenthal (1978)) and, thus, immune from even the largest observed shifts in the gridlock interval.
to the world around it worsens.\footnote{It is, of course, possible that the fit may improve over time, although it is not clear why a carefully designed policy at time \( t \) may be improved upon by the whims of chance. Given the expected infrequency of this possibility in practice, we leave it aside for the moment. In any event, this possibility does not substantively alter any of our results, effectively only slowing down the rate of decay.}

We introduce policy decay into a simple, paradigmatic model of legislative policymaking and show that it leads to a starkly different perspective on politics. Our results upend the classic notion of gridlock, and bear implications more broadly for the practice of politics. We show in turn how a changing world impacts the power of agenda control, how it drives the dynamic path of legislation, how it reveals a novel yet empirically relevant notion of expertise, and how it, ultimately, provides a foundational logic to the design of bureaucracy.

**DECAY IN PRACTICE**

It is self-evident that the United States today barely resembles the United States of 1789. Change has come in the form of surprising, profound, and dramatic innovations, like the harnessing of nuclear power, or the invention of the automobile and the internet. Change has also come in the form of mundane, incremental change. Indeed, it is difficult to identify an aspect of society that has not changed significantly over time. All of this change has impacted the fit of policy with its environment, rendering inefficient even the most carefully crafted policies.

The more substantial changes, and their impact on policy, have provided the basis for much scholarship in political science. A classic example is Skowronek (1982)’s account of the changes wrought on the American economy by the industrial revolution. Skowronek showed how the shock of industrialization left the original institutions of the founding era essentially incapable of effective regulation. This decay in policy effectiveness necessitated a transformative state-building effort to restore the federal government’s capacity to govern effectively.

Similar ideas are evident in the work of Hacker and Pierson (2010). Their book-length treatment offers numerous, contextually rich descriptions of policies that drift from their original intent, to use their terminology. Hacker and Pierson focus on change that is distributional, moving the outcome of policy either left or right over time, but always delivering
ideal policy to some citizens. For instance, they show how the carried interest provision in the tax code ultimately came to benefit one set of interests—private equity and hedge fund managers—that were very distant from the original target. In this paper we instead focus on change that benefits no one—change that is Pareto inefficient—and to distinguish these concepts we introduce the alternative terminology of policy decay.\footnote{Although decay and drift are distinct effects, the same underlying force resonates. As Hacker and Pierson (2010) (p. 170) describe: “Drift describes the politically driven failure of public policies to adapt to the shifting realities of a dynamic economy and society. Drift is not the same as simple inaction. Rather, it occurs when the effects of public policies change substantially due to shifts in the surrounding economic or social context ...”.
}

As these examples attest, decay is difficult to predict and often surprising, and is best thought of as a random process, with decay arriving irregularly and in different degrees. Even for aspects of society where change is predictable—such as inflation or population size—the rate of change itself is noisy and imperfectly predictable. And for many forms of change—such as profound discoveries or changing tastes—the nature of change is by definition unpredictable.

The richness of decay, in both scale and timing, implies that observing decay is no trivial matter. It follows, therefore, that not all policymakers observe decay simultaneously. Information about a changing environment is often scattered throughout society, albeit with a concentration among interests most directly affected. Consequently, policymakers closer to these interests gain an advantage in policy negotiations over their less informed colleagues. In most political systems, including that in the United States, it is specialist bureaucrats who are closer to the ground on any particular issue than are generalist legislators.\footnote{One may see this effect more directly when it comes to government statistics (like inflation, unemployment, housing starts, etc.) that are literally collected and compiled within the executive branch.} Indeed, as detailed by Gailmard and Patty (2012), the acquisition of information from regulated interests is often an explicit objective in the design of executive branch agencies. Decay thus gives rise to a type of policy expertise—a temporal expertise—that is distinct from canonical conceptions. It is an expertise that emerges from the time lag at which new information is learned rather than some deeper understanding of the policy mapping itself.
OVERVIEW OF RESULTS

In this paper we provide a formalization of decay and explore how it impacts our conception of policymaking. We conceive of decay as lowering the quality of policy, imposing an inefficiency on all policymakers, be they liberal or conservative. The payoff from this perspective is immediate. Our first result is simple yet substantively striking. We show that in the presence of decay the classic notion of policy gridlock breaks down. Every policy, including those in the gridlock interval, is up for grabs and subject to change. Decay provides a unifying force that can overcome persistent ideological disagreements such that even the most centrist status quo can be defeated.

That policy can change is only the first part of the story. The next question is how policy changes. We begin our analysis with a benchmark model in which decay arrives smoothly and consistently, with all other political frictions set aside for clarity. In this setting we show that decay and legislative action possess a symbiotic relationship. Whenever decay appears, it is immediately and completely removed by passage of a new statute. Even a hint of decay is sufficient to break down ideological disagreements and generate legislative action. Yet the gains from this legislative action are not equally split. Although decay affects all policymakers equally, it provides a new, and powerful, leverage to the holder of agenda control. Consequently, a strategic and self-interested agenda setter can use the leverage from decay to extract policy concessions from otherwise recalcitrant legislators.

As this process iterates, the path of policy traverses a path of continuous change, slowly yet inexorably converging on the ideal point of the agenda setter, even when policy is nominally within the classic gridlock interval. To an outside observer, this behavior may seem puzzling as it appears that legislators acquiesce repeatedly to the demands of the agenda setter, seemingly without cause, violating the standard intuitions of gridlock. We show that with decay—specifically the looming prospect of decay—this behavior makes perfect sense. In providing a new interpretation for this type of behavior, our model sheds new light on many curious episodes of policymaking in practice.

For theorists of policymaking, the challenge, typically, is to explain policy change. Our benchmark model turns this problem on its head. Policymaking not only occurs in equilibrium, but it occurs at every point when decay emerges. In a sense, the benchmark model
delivers too much policymaking. As Hall (1993) argues in a different but related context, the challenge then becomes explaining the causes of policy stability in addition to the causes of policy change.

To this end, we explore the role of expertise, namely *temporal* expertise when one policymaker observes decay earlier and before policymaking takes place. A temporal expertise only makes sense when decay arrives stochastically, and to this end we suppose the arrival time and degree of decay is random. To emphasize the importance of expertise, we demonstrate that stochastic decay alone is insufficient to alter the conclusions of the benchmark model.\(^6\) When one legislator holds a temporal expertise, however, political behavior and policy outcomes are upended. Legislative deals are rejected in equilibrium, with decay being left to fester and work its inefficiency, accumulating over time. The policy path now exhibits periods of inactivity followed by more substantial change, a pattern that resonates with the concept of “punctuated equilibrium” of Baumgartner and Jones (1993) and Hall (1993), although with an underlying logic that is very different.

Ironically, we find that decay is more likely to accumulate and grow when it arrives slowly. Sudden crises, caused by the arrival of large blocks of decay, engender immediate policy responses, whereas, in contrast, a slow boiling crisis—like the proverbial frog in a pot of boiling water—is left by policymakers to fester and grow, reaching greater depths before a policy response occurs.

The inclusion of decay brings to the forefront the question of efficiency. Policymaking no longer determines only whether a policy is left or right, but also whether it is implemented effectively. Our benchmark model shows that it is possible in equilibrium for policy to be implemented efficiently—without decay—at every point in time. Although this property breaks down in the presence of other frictions, it represents a remarkable ability of basic legislative processes to push legislators toward effective agreement. This represents only a weak, *static* form of efficiency, however, and does not speak to whether the policy path is *dynamically* efficient. Unfortunately, performance on this dimension is less sanguine, and in equilibrium the policy path is dynamically inefficient, even in the benchmark model. We show that legislative deals exist that makes all policymakers better off over time, but

\(^6\)The only impact being that the rate of policy change now reflects the stochastic rather than smooth arrival of decay.
that such deals are not supportable in equilibrium. This result reveals a basic, dynamic limitation of political institutions, reinforcing the importance of adopting a dynamic view of policymaking.

Temporal expertise is not the only feature of practical policymaking that can slow down the responsiveness of policymaking. We consider several other possibilities here, the most notable of which is settings where some policies can be implemented with greater efficiency than others. We show that this induces a policy path that is both statically and dynamically inefficient, even in the absence of other frictions. Policymakers now reject deals that improve everyone’s utility at that point in time. The logic for this decision captures the essence of forward looking strategic behavior: Voting yes will improve a legislator’s immediate welfare but it will trigger a path—a slippery slope—that ultimately leaves her worse off.

2 A Dynamic Model of Policy Decay

We develop a reduced form model of legislative policymaking between two key policymakers: a Proposer $P$ and a pivotal Voter $V$. At each time $t$ the two players bargain over policy. The bargaining process is a straightforward ultimatum game in the style of Romer and Rosenthal’s (1978) setter model. The Proposer makes a take-it-or-leave-it policy proposal and the Voter either accepts or rejects the proposal. If she accepts then the proposal becomes law, and if she rejects the status quo statute remains in place.7

The novelty in our modeling is in two key departures from Romer-Rosenthal. First, we do not limit attention to a one-shot interaction, analyzing instead a fully dynamic model. Second, we allow the quality of policy to decay over time. We formalize these two features in what follows.

The policy space is two dimensional. The first dimension is the standard left-right continuum, commonly referred to as the ideological dimension. The second dimension is a quality dimension. For a fixed ideological position, all citizens prefer higher quality to lower quality. The quality dimension represents a public good, whereas preferences diverge on the ideological dimension. A policy is denoted by $(x_t, q_t)$, where $x_t$ denotes the ideological location and

---

7 As in the standard formulation, we assume that the Voter’s tie-breaking rule is to vote in favor of a proposal when indifferent between the proposal and the status quo.
$q$, the quality level. We fix the maximum quality to be 0, and normalize the Voter’s ideal point in the ideological dimension to be 0 and the Proposer’s ideal point to $\pi > 0$. Figure 2 depicts the policy space and ideal points of the two players.

It is possible in this setting for two policies to share an ideological location but differ in quality, and the policy with the higher quality will Pareto dominate the other. The set of undominated policies constitutes the *efficient frontier* of policy. This frontier consists of all policies with maximum quality (i.e., zero quality) and is represented by the horizontal axis in Figure 2 and is where the policymaker ideal points are located. The classic gridlock interval is then the interval on this frontier between the ideal points of the two political actors, $[0, \pi]$, as it is for these policies alone that an alternative doesn’t exist that makes both policymakers better off.

The policymakers have preferences that are represented by differentiable, quasiconcave utility functions. Utility at time $t$ is given by $u_{V,t}(x,q)$ and $u_{P,t}(x,q)$ for the Voter and Proposer, respectively. For convenience and without loss of generality, we normalize utility to zero at their ideal points, $u_{V,t}(0,0) = 0$ and $u_{P,t}(\pi, 0) = 0$. For our main results we require only that utility is strictly decreasing in ideological distance and increasing in quality.\(^8\) For clarity, we specialize utility at several points to be quasilinear and given by the functional

\[^8\text{That is, } \frac{du_{V,t}(x,q)}{dq} > 0 \text{ and } \frac{du_{P,t}(x,q)}{dq} > 0, \text{ and } \frac{du_{V,t}(x,q)}{d|x|} < 0 \text{ and } \frac{du_{P,t}(x,q)}{d|x-\pi|} < 0.\]
form:
\[ u_{V,t}(x_t, q_t) = -\alpha_{V} x_t^2 + q_t, \quad \text{and} \quad u_{P,t}(x, q) = -\alpha_{P}(x_t - \pi)^2 + q_t. \quad (1) \]

To capture dynamics in policy making, we allow time to continue indefinitely. In discrete time, this means that policymakers bargain – and receive utility – at each time \( t = 1, 2, 3, \ldots \). Policymakers discount the future at per-period rate \( \delta \), where \( \delta \in [0, \infty) \). The limiting cases are \( \delta = 0 \), which implies perfectly far-sighted players who do not discount the future at all, and \( \delta = \infty \), which implies perfectly myopic players who care only about the utility at the present instant. Total utility is given by:

\[ u_{V} = \sum_{t=1}^{\infty} \left( \frac{1}{1+\delta} \right)^{t-1} u_{V,t} \quad \text{and} \quad u_{P} = \sum_{t=1}^{\infty} \left( \frac{1}{1+\delta} \right)^{t-1} u_{P,t}. \]

Although discrete time is the most commonly used approach to modeling dynamics, it has limitations. It restricts exogenously when policymakers can choose to bargain and change policy, and the indivisibilities it gives rise to can prove analytically clunky. To improve the clarity of our results and to allow policymakers to choose both when as well as what policies to implement, we also consider behavior when time varies continuously. Specifically, at each instant \( t \in [0, \infty) \) policy can be changed and the policymakers experience the utility of the policy that is in place.\(^9\) Discounting remains at the same rate \( \delta \), although the expression for total utility now yields an integral rather than the more familiar summation.

\[ u_{V} = \int_{0}^{\infty} e^{-\delta t} \ u_{V,t} \ dt \quad \text{and} \quad u_{P} = \int_{0}^{\infty} e^{-\delta t} \ u_{P,t} \ dt. \]

To capture the idea of policy decay, we suppose that the quality of policy worsens as time passes. Decay is exogenous to the policymaking process, reflecting changes in the surrounding world. Demographics change, new technologies are invented, tastes evolve, and all of these lead to the fit of policy to the external world to worsen.\(^10\) For simplicity, we begin in the baseline model with the assumption that decay is deterministic and inexorable.

\(^9\)Although rare in political science, continuous time more accurately captures the freedom of policymakers to use time as a strategic variable. See Dewan and Myatt (2012, 2010) for elegant exceptions that utilize the richness of continuous time.

\(^10\)A changing world that occasionally improves the fit of policy effectively only slowing down the rate of decay and does not change our results. For this reason, and because decay resonates more empirically, we do not model improvement formally.
Specifically, policy quality decays at the constant rate \( \lambda \). This means that if no legislative action is taken for a length of time \( \hat{t} \), the resulting quality will be \( \lambda \hat{t} \) lower (whether time is discrete or continuous).\(^{11}\)

We set the initial status quo policy to the Voter's ideal point of \((0, 0)\). Thus, policy begins within the classic gridlock interval. Immediately thereafter policy begins to decay at rate \( \lambda \). At each time \( t \), for a status quo policy \((x^s, q^s)\), the policymaking process proceeds as follows:\(^{12}\)

**Timing at each \( t \)**

1. Decay is realized and observed by both players (but not yet experienced).
2. Proposer offers an alternative \((x, q)\).
3. Voter accepts \((x, q)\) or rejects and gets \((x^s, q^s - \lambda \Delta t)\).
4. Payoffs realized and the implemented policy becomes the status quo.

To fix ideas we make the assumption that decay can be removed by legislation before it affects utility. The alternative assumption (that decay is experienced as soon as it appears) does not substantively affect our results, although it makes the description of the equilibrium path analytically messy.\(^{13}\) Our formulation implies that policymakers can remain on the efficient frontier if they wish, and, thus, that any departure from the efficient frontier constitutes a political failure.

The model just described constitutes our baseline formulation. We have deliberately kept it simple so as to make the effect of decay on policymaking most transparent. The reality of policymaking is considerably more complex, with numerous frictions, institutional variants, and additional participants. After analyzing the baseline model and the core effect of decay, we enrich the model in various directions to better understand how decay manifests in policymaking. We leave the details of these extensions until required.

Throughout the paper we are (mostly) agnostic as to how the identity of the players are

---

\(^{11}\)Formally, of course, the policy itself remains unchanged, yet the outcome it produces is now different. In the interests of simplicity, we have minimized the description of the separate choice of policy and the mapping from policies to outcomes, focusing exclusively on the outcome space.

\(^{12}\)\(\Delta t\) is the time that elapses between periods; in the continuous-time limit this becomes an infinitesimally small time interval \( dt \).

\(^{13}\)In continuous time this distinction is without a difference and the equilibrium paths are identical.
Figure 2: The leverage granted to the Proposer by the appearance of decay. The dashed line represents the Voter’s indifference curve.

interpreted. We do adopt the standard interpretation that the game between the Proposer and Voter is a reduced form representation of a larger bargaining game within and between branches of government, with the Proposer shorthand for the (possibly aggregate) preferences of the agenda setter, and the Voter representing the pivotal voter, whomever may fill that role in practice. Our favored interpretation is that these roles are filled by Congress and the President, respectively, although it is possible that in some instance – such as budget politics – these roles are flipped.

3 Equilibrium Decay in the Baseline Model

To see the impact of decay, consider a situation where the status quo policy $x'$ has decayed by amount $\lambda$ to position $(x', -\lambda)$, as depicted in Figure 2. The pure inefficiency of decay offers the prospect for mutual benefit. The Proposer could offer to update policy and return to its starting point, removing decay without changing the ideological location, such that decay may as well never have happened.

In politics, however, we must always distinguish what is possible from what will happen in equilibrium. That is still the case here. The Proposer can make both policymakers better off by returning to $(x', 0)$, but he can make himself even better off by offering a return to the efficient frontier that pushes toward his own ideal point, as depicted by policy $x''$. The voter is then left with a dilemma: vote yes and concede ground on ideology, or vote no, holding
ideological ground but enduring a decayed policy. As long as the Proposer doesn’t overreach on ideology, the Voter will be better off voting yes. The Voter will accept, so long as the offer falls to the left of her indifference curve with the status quo \((x', -\lambda)\), depicted in the figure as a dashed line.

This logic is easy to grasp, yet a subtle implication is that decay is not actually observed. To a naive observer, it would appear that the Voter is giving up ground to a dominant Proposer—perhaps a President acquiescing to a forceful Congress—despite policy beginning within the gridlock interval and seeming unassailable. Clearly, however, decay is the driver of this policy change even though it never materializes. The looming prospect of decay provides the Proposer with a threat point, and this threat point allows him to leverage his monopoly agenda power to win policy concessions. These concessions are made by a rational, strategic actor, and there is no role for a legislator’s force of will or his character.

In a one-period game the equilibrium behavior follows this reasoning: The Proposer, applying the familiar logic of take-it-or-leave-it bargaining, offers a policy on the efficient frontier that maximizes his own utility while leaving the Voter indifferent between voting yes or no.

Yet policymaking in practice—as in our game—is dynamic and we seek to understand some deeper questions. How does this logic iterate over time? Is decay always removed whenever it looms such that efficiency is maintained or does it accumulate on the equilibrium path? How does the Proposer leverage the recurring prospect of decay for maximum advantage? And, ultimately, what is the dynamic path of policy over time? The answers to these questions depend on several factors, although general and robust insights do emerge. One determinative factor is the planning horizon of the policymakers. To expose the underlying intuitions most clearly, we break out our results according to whether policymakers look to the future or focus only on the present in their decision making. We begin with the case of short-sighted policymakers. We focus on the underlying intuition in describing the results and, for brevity, relegate all formal proofs to the Appendix.
3.1 Short-Sighted Policymakers

The pressures of elections can cause policymakers to focus excessively on the present as they may not be around tomorrow. We begin with this case as it is both realistic and analytically more transparent. We push the present-bias to the maximum, setting $\delta = \infty$ so that policymakers care solely about the current period.

The logic of this case follows directly from iterating the one-shot logic described above. At each point in time, decay appears and both policymakers prefer to move back to the efficient frontier. The Voter agrees to anything that leaves her as well off as she would be if decay were left in place, and the Proposer, also being shortsighted, offers the maximum ideological shift that he can get away with. Neither player looks to the future to see how behavior in one period will change the default status quo in the next.

This iteration of this dynamic is depicted in the left panel of Figure 3. Policy leapfrogs across the policy space until it arrives in the neighborhood of the Proposer’s ideal point, after which it jumps directly to the Proposer’s ideal, and then returns there every period thereafter. The size of the jump in each period depends on parameters of the model, including the length of the time period. Holding all else constant, the right-side panel of Figure 3 depicts the dynamic path as the length of time between periods is halved. Decay is smaller in each period (as less time has passed) and the size of the ideological jump is correspondingly smaller too.

As the period length gets smaller and smaller, the opportunities for policymaking become
more and more frequent, and we approach the limit of continuous time. In the limit, decay occurs at every instant, and in every instant a new policy is proposed and accepted that removes this decay and returns policy to the efficient frontier. And at every instant, with every new policy, the ideological location edges slowly but inexorably toward the Proposer’s ideal point, where it remains thereafter.

The resulting path of policy over time can be stated explicitly, as a function of the parameters of the model. For clarity and simplicity, we state equilibrium behavior in Proposition 1 for the limit case of continuous time.

**Proposition 1** Consider the continuous-time version of the game described in Section 2. Suppose that the marginal cost of decay to the voter along the efficient frontier is strictly positive, i.e. \( \frac{\partial u_{V,t}}{\partial q} \big|_{q=0} > 0 \). Then, in the unique subgame perfect equilibrium with short-sighted policymakers a proposal \((x_t, q_t)\) is offered at each \(t\) and accepted, where:

\[
\begin{align*}
x(t) &= \min \left[ u_{V,t,0}^{-1} \left( -\lambda \int_0^t \frac{\partial u_{V,t}}{\partial q} \, dt' \right), \pi \right] \\
q(t) &= 0
\end{align*}
\]

Where \(u_{V,t,0}\) is the slice of the Voter’s utility function holding \(q\) constant at 0.

In words, the equilibrium path slides along the efficient frontier at a rate that is determined by the Voter’s relative valuation of ideological location versus quality. Decay does not appear in equilibrium. Because every offer leaves her exactly indifferent, the Voter’s utility along the equilibrium path changes at exactly the rate at which it would be changing if she rejected the offer and allowed decay to occur. The more relative weight does the voter place on ideology, therefore, the slower will movement be.

The equilibrium dynamic in the proposition is striking yet clearly extreme. It should not be taken as a positive description of practice. The proposition provides a prediction of what policymaking should look like in a frictionless model of policymaking with decay. It highlights that decay need not manifest to be important to politics as the prospect of decay is alone sufficient. The proposition also highlights how the classic notion of the gridlock interval does not survive contact with a world that changes, evolves, and decays. In subsequent results
we bring the equilibrium predictions closer to practice by adding frictions to the model of policymaking, yet this basic force always persists.

The path described in Proposition 1 simplifies significantly if the Voter’s utility is quasi-linear (Equation (1)). To illuminate the equilibrium behavior, we restate utility for this case.

**Corollary 1** Suppose the voter’s instantaneous utility is quasilinear: \( u_{V,t}(x,q) = f(x) + q \) for some function \( f \). In the unique subgame perfect equilibrium when players are myopic, a proposal \((x_t,q_t)\) is offered at each \( t \) and accepted, where:

\[
    x_t = \min \left[ f^{-1}(-\lambda t), \pi \right] \\
    q_t = 0
\]

For example, when the Voter has quadratic-loss utility over the ideological dimension \((f(x) = -\alpha_V x^2)\), the path is:

\[
    x_t = \min \left( \sqrt{\frac{\lambda t}{\alpha_V}}, \pi \right)
\]

This special case allows us to specify the equilibrium policy trajectory precisely. This is depicted in Figure 4. Policy begins at the Voter’s ideal point and moves inexorably across the efficient frontier toward the Proposer’s ideal point. At first policy movement is rapid, but over time it slows down. The change in speed comes from the shape of the utility function, specifically the utility function of the Voter. As it is the Voter who is driven to indifference each period, it is how she makes the trade-off between accepting decay and moving back the efficient frontier that determines the rate of policy movement. With quadratic-loss over the ideological dimension, initial increments from her ideal point are not so costly to the Voter and she is willing to make substantial concessions to remove decay. As time goes on, however, and as policy moves further from her ideal point, the Voter increasingly dislikes further ideological concessions. As the cost of decay is constant throughout this process (as utility is quasi-linear), the rate of movement slows as policy gets closer to the Proposer’s ideal point.
3.2 Far-Sighted Policymakers

A short-sighted Voter concedes on policy each period, only for a fresh round of decay to emerge and leaving her exposed, again willing to make policy concessions. In practice policymakers are not so myopic, often strategizing about plans well into the future (Moe, 2013). This suggests the possibility that a far-sighted Voter ($\delta < \infty$) will anticipate a recurrence of her predicament should she concede on policy, and reject the Proposer’s offers. Or perhaps it is the Proposer who will hold out, deliberately and strategically choosing to not make any offer to the Voter so that decay festers and increases his leverage to extract even larger concessions in the future.

Our next result shows that, surprisingly, neither of these possibilities emerge in equilibrium, even for far-sighted policymakers. In fact, the equilibrium policy trajectory is qualitatively similar to that when policymakers are short-sighted:

**Proposition 2** Suppose time is continuous and policymakers far-sighted ($\delta < \infty$). There exists a unique subgame perfect equilibrium in which policy is updated continuously: At every instant, the Proposer offers a new policy that returns policy to the efficient frontier and the Voter accepts, and the policy trajectory moves continuously toward the Proposer’s ideal point,
where it remains thereafter.

Although the equilibrium resembles that of the short-sighted case, it is not that far-sighted calculations are irrelevant. Indeed, it is exactly the calculus of how holding out will improve or undermine a policymaker’s bargaining position that determines the rate of policy change in equilibrium. That in equilibrium these threats are not carried out reflects less on their importance and more on the ability of policymakers to anticipate them and to allow for them in bargaining over policy.

To see how far-sighted considerations matter for policymaking, consider again the special case of quasi-linear utility. Surprisingly, in this case—and only in this case—the equilibrium path is exactly the same for far-sighted policymakers as it is when policymakers are short-sighted.

**Proposition 3** Suppose the voter’s instantaneous utility is quasilinear: \( u_{V,t}(x, q) = f(x) + q \) for some function \( f \). In the unique subgame perfect equilibrium when players are forward-looking, a proposal \((x_t, q_t)\) is offered at each \( t \) and accepted, where \((x_t, q_t)\) are the same as those given in Corollary 1. This is the only case where such equivalence holds.

To see why this is the case, note that quasi-linearity implies that the cost of decay is everywhere the same, on the efficient frontier and off it. Thus, should the Voter reject an initial offer, the cost of decay in the next instant will be exactly the same as if she’d voted yes and returned to the efficient frontier. She is then ready after two doses of decay to accede to the exact same policy concession as she would have conceded to in aggregate had she accepted, returned to the frontier, and faced a second dose of decay.

Quasilinearity implies that the Voter’s relative bargaining position is the same regardless of where policy is in the space, and that looking down the game tree does not alter the Voter’s calculus. The logic of this result also reveals why it breaks down for other forms of utility. If the cost of decay to the Voter does change—be it an increase or decrease—then her bargaining position will change, and this will change both her willingness to accept proposals and, consequently, the policies that the Proposer offers in equilibrium.

If the Voter’s marginal cost of decay increases in the distance from the frontier, then the Voter’s bargaining position will deteriorate should decay persist. Anticipating this, and
knowing that he can extract greater policy concessions should decay grow, the Proposer will make greater policy demands initially. Anticipating the same outcome, the Voter is more willing to concede on policy initially. As a result, the policy trajectory moves at greater speed and arrives at the Proposer’s ideal point earlier. On the other hand, if the Voter’s marginal cost of decay decreases away from the frontier then the logic is reversed. The Proposer anticipates that he will need to make smaller demands in the future should decay grow. The Voter knows this as well, so the Proposer tempers his demands today and policy traverses the efficient frontier more slowly.

We summarize these comparative statics in Corollary 2.

**Corollary 2** If the Voter’s marginal cost of decay is increasing as distance from the frontier increases, the rate of policy movement in equilibrium is faster than the myopic benchmark. Conversely, if the Voter’s marginal cost of decay is decreasing as distance from the frontier increases, the rate of policy movement in equilibrium is slower than the myopic benchmark.

The results of this section reveal the rich impact that strategic, far-sighted planning can have on policymaking. Although in equilibrium the policy path never leaves the efficient frontier, the costs of delay and decay well of this path are still fundamental to behavior as they provide the basis for off-equilibrium-path threats. Far-sighted policymakers, seeing these threats and anticipating their consequences, allow for them and respond accordingly. The upshot of these calculations is that decay is removed instantly, yet the rate at which the Voter concedes on policy for this to happen is determined by how rapidly the Voter expects her bargaining position would deteriorate if she were to allow decay to accumulate.

The equilibrium logic just described focuses exclusively on the Voter’s preference over decay. The reader may wonder why it is not relevant that the Voter, herself, can’t hold out and force a better deal from the Proposer should he face a larger cost of decay. After all, when driving the Voter to indifference, the Proposer is far from indifferent over whether the Voter accepts or not. Unfortunately for the Voter, there is a limit to the credibility of her threats, and it is here that subgame perfection binds, giving closure to our equilibrium.

To see this limit, suppose the Voter were to reject all offers and allow policy to decay. Initially, the Voter is indifferent between this path and accepting the Proposer’s offers, whereas
the Proposer is significantly worse off. Ultimately, however, a point \((0, -q^*_t)\) is reached at which the Voter is indifferent between that point and the Proposer’s ideal point \((\pi, 0)\).

On the equilibrium path, policy gets no worse for the Voter once \((\pi, 0)\) is reached. As the Proposer desires no greater policy concession than his own ideal point, once this point is reached the policy will remain there forever after. By contrast, if she continues to reject offers beyond \((0, -q^*_t)\), further decay arrives and the Voter’s utility continues to decrease without bound. Consequently, the Voter will surely accept the offer of \((\pi, 0)\) if the status quo is \((0, -q^*_t)\); any threat to hold out ceases to be credible beyond this point. Anticipating this boundary, the Proposer can make an offer at the instant just before \(-q^*_t\) is reached that he knows the Voter will certainly accept. Iterating this logic unravels the Voter’s threat all the way back to the frontier, implying that there is always some offer the Proposer can make which forestalls decay and which the Voter will accept.

### 3.3 Static versus Dynamic Efficiency

A remarkable feature of equilibrium behavior is that the policymakers are always able to work out their differences. Despite the threat of inefficiency from decay, and despite the important role it plays in actual policy choice, the policy implemented is never actually inefficient. This suggests an impressive ability of the policymakers, and the institution in which they operate, to reach consensus efficiently. Yet that is not the complete story and hides a deeper inefficiency that is not worked out within bargaining.

The ability of policymakers to remain on the efficient frontier represents static efficiency. At each point in time policy is efficient. However, the fact that over time policy traverses across the efficient frontier implies that it is dynamically inefficient. To see this inefficiency, consider the classic case of policymakers having quadratic-loss utility over the ideological space and being risk averse. A fundamental property of risk aversion is a preference for a constant policy over one with the same expected value but with positive variance.

In practice, this implies that there is a point on the efficient frontier that, should it be implemented in every period, would leave both the Proposer and the Voter better off than they are in equilibrium. That such a stable policy is not supportable in equilibrium exposes a shortcoming in the underlying political institution to resolve differences. Note that because
The inefficiency is purely dynamic, this shortcoming would have been missed in a static, snapshot view of the policymaking process.

The reason why policymakers can attain static but not dynamic efficient outcomes turns on their ability (and inability) to forge deals statically versus dynamically. Static efficiency demands only an immediate transaction of costs and benefits between the players (i.e., removal of decay versus ideological concessions). In contrast, dynamic efficiency requires that the policymakers engage in a dynamic transaction, with some costs paid now and some benefits received later. To implement a constant policy outcome across time, the Voter would need to make larger ideological concessions initially and be compensated later with policy not moving further toward the Proposer’s ideal point. The Achilles heel of this intertemporal trade-off is that the Proposer has the incentive to renege after the Voter’s initial concession. Decay will continue to arrive and the Proposer can use it to leverage further concessions from the Voter. Anticipating this, the Voter refuses to make her initial concessions, and the dynamically efficient compromise falls apart.

This logic exposes the shortcoming of a political institution built on short-term deals. Being unable to tie their hands in the future, the policymakers face a hold-up problem that prevents long-term agreements—even when efficient—from being executed. This inability is not merely a pathology of our parsimonious model. It is an important principle of democracy—and is enshrined in the U.S. Constitution—that a sitting of Congress cannot tie the hands of future Congresses. While this principle delivers its own benefits, it does constrain dealmaking that requires a longer horizon to play out. This need not be the end of the story, however. It is also a long-standing principle that institutions evolve to serve the needs of the populace, and our results suggest a need for U.S. political institutions to evolve to accommodate longer-term deal making. We take up this issue when we consider the role of the bureaucracy and bureaucratic design in the discussion section.

4 Frictions, Imperfections, and Decay in Practice

The typical challenge in formal models of policymaking is to explain why and how policy changes from the status quo. The benchmark model turns this problem on its head. Policy not only changes but it changes continuously. The theoretical challenge then becomes
explaining the periods of stability, to explain what causes policy to sometimes be stable and other times in flux. We proceed by introducing frictions into the policymaking process. The first friction is the most simple: an exogenous cost to make policy proposals. This cost generates periods of stability and the accumulation of decay on the equilibrium path, yet overall delivers an incomplete account of policymaking. To get at the richness of possibilities evident in practice, we then allow for an efficient frontier that is not linear, before allowing for the emergence of temporal expertise, our main focus and the subject of Section 5.

4.1 Proposal Costs

In practice, proposing a new policy, having it accepted, and implementing it, are not cost-free. Suppose that a fixed cost is incurred by the Proposer every time he makes a proposal and let this cost be \( c > 0 \). Clearly, this cost renders the equilibrium paths in the previous section infeasible, as these paths involve continuous updating of the status quo. In the presence of proposal costs, changing policy continuously will incur an infinite cost that swamps the marginal cost of decay. Consequently, and quite realistically, the presence of a proposal cost will slow down the policymaking process, meaning that decay will appear and persist, at least for some time, along the equilibrium path.

The amount of time that decay persists depends, in a straightforward way, on the magnitude of the proposal cost \( c \). Because the cost is paid by the Proposer, nothing about the Voter’s acceptance rule changes compared to the baseline case. However, the addition of the cost changes the Proposer’s calculus: it must be worthwhile to make an offer and pay the cost now, rather than waiting, deferring the cost and experiencing further decay, but extracting a larger concession from the Voter. This trade-off rule yields the following decision rule.

**Remark 1** When the current state is \((x_0, q_0)\), where \( q_0 < 0 \), and proposal costs are \( c > 0 \), the Proposer makes an offer if and only if:

\[
\frac{u_{P,t}(x^*, 0) - u_{P,t}(x_0, q_0)}{\delta} > c,
\]

where \((x^*, 0)\) is the point on the efficient frontier that satisfies the Voter’s instantaneous indifference condition with \((x_0, q_0)\).
As \( x^* \) is increasing in the time that has been allowed to elapse with no policy change, larger costs imply longer periods in which decay takes hold. For large \( c \), the equilibrium policy trajectory exhibits a single period of decay followed by a movement to \((\pi, 0)\), with periodic updating to remove accumulated decay thereafter; for small \( c \) the trajectory exhibits several intermediate jumps to ideological points in the interval \((0, \pi)\) before reaching the steady state.

![Diagram](image)

**Figure 5:** An example equilibrium path with nonzero proposal costs. In the case depicted here, both players’ utility functions are quasilinear in quality with quadratic loss over the ideological dimension. In the left panel, costs are low enough that the Proposer makes several intermediate offers before reaching the steady state, in which the ideological location remains constant but decay accumulates and is periodically removed. In the right panel, costs are high enough that the Proposer waits until he can achieve his ideal ideological location with one proposal.

Figure 5 depicts these two cases visually. In the case where proposal costs are relatively low, the time between proposals gets longer (and thus, the amount of decay that is allowed to accumulate before there is legislative action increases) as the policy gets closer to the Proposer’s ideal.\(^\text{14}\) Over time, the rate of legislative activity slows steadily, and the amount

\(^{14}\text{This feature is a consequence of the concavity of the proposer’s utility function on the ideological di-}
of decay that is experienced steadily increases. In the high-cost case the transition is abrupt rather than gradual, but the pattern is the same: action initially occurs relatively quickly, but slows once the Proposer’s ideal is reached and no further ideological gains are possible.

### 4.2 A Variable Efficient Frontier

We have assumed so far that all policies, when working well, achieve exactly the same level of efficiency (normalized to zero quality). That is, it is possible to devise an equally efficient policy using any set of ideologically-appropriate tools. In practice, however, some policy areas may well be more amenable to certain ideological approaches than to others. To give just one example: In the recent debate over healthcare reform, several single-payer alternatives proposed by liberal Democrats were projected to have lower aggregate costs than the proposal preferred by moderates that retained the participation of private insurers. Such differences may also emerge from what Volden and Wiseman (2014) refer to as legislative effectiveness, as proponents of certain policies are more effective than others at crafting efficient legislation.\(^\text{15}\)

To capture this possibility in its most stark form, suppose the efficient frontier is as depicted in Figure 4.2 (and again set proposal costs to zero). In this case, all policies with the exception of \(\hat{x}\) are equally efficient, reaching a maximum at 0, whereas \(\hat{x}\) is significantly more efficient at its best, potentially attaining an efficiency level of \(\hat{q} > 0\). Suppose further that \(\hat{q}\) is sufficiently high that the Voter strictly prefers policy \(\hat{x}\) with efficiency \(\hat{q}\) than she does her own ideal point at its maximum efficiency of zero; i.e., that \(u_{V,t}(\hat{x}, \hat{q}) > u_{V,t}(0, 0)\), as represented by the dashed indifference curve for the Voter.

The import of this set-up is that the Voter’s ideal point, and many other points, are no longer part of the efficient frontier as they are dominated by \((\hat{x}, \hat{q})\). Thus, it is a strict Pareto improvement if policy jumps immediately at the beginning of play from \((0, 0)\) to \((\hat{x}, \hat{q})\). The previous finding that policymakers always attained static efficiency implies, therefore, that

\(^\text{15}\)Note that some situations like this (with a variable efficient frontier) can be accommodated in our baseline (flat frontier) model by transforming the utility function over the ideological dimension appropriately. It is only when the efficiency differences are so great that the resulting transformed utilities are no longer single-peaked that the flat frontier is a substantive restriction.
this should be the first policy change in equilibrium. If, indeed, this were a one-shot game, or if policymakers were short-sighted, then this is exactly what would happen as it maximizes both players’ utility in the short run. Our next result, however, shows that with far-sighted policymakers this does not occur in equilibrium. Thus, with a variable efficient frontier, the ability of policymakers to find efficient static bargains breaks down.

**Remark 2** Suppose there exists some \( \hat{x} > 0 \) where the maximum efficiency is \( \hat{q} > 0 \), as described above, and \( u_{V,t}(\hat{x}, \hat{q}) > u_{V,t}(0, 0) \), i.e., the statically efficient policy is to immediately jump to \( (\hat{x}, \hat{q}) \). For intermediate values of \( \delta \in (0, \infty) \), the Voter may reject an initial offer of \( (\hat{x}, \hat{q}) \).

The unwillingness of the Voter to move to policy \( (\hat{x}, \hat{q}) \) is not because the Voter does not recognize its superiority; indeed, this is why she would agree to the proposal were she short-sighted. Rather, the Voter refuses to agree to change inefficient policies as she anticipates what would come next should policy jump to \( (\hat{x}, \hat{q}) \). This is a novel twist on the idea of a ‘slippery slope.’ Should she agree to the jump, decay would continue to appear and the
Proposer would continue to exploit it to further policy movement toward his own ideal point (which he prefers to \((\hat{x}, \hat{q})\)). The Voter anticipates that agreeing to an efficient policy change in the short-term would only hasten the slide to less ideologically attractive policies in the longer term. It is fear of this slippery slope that causes the voter to veto Pareto improving policy changes.

5 Stochastic Decay

Change does not always arrive in a steady stream. When it arrives, as well as the form it takes, often is difficult to predict. In this section we allow for decay to arrive stochastically. We amend the timing at each \(t\) by amending Step 1 in the following way:

**Step 1—amended:** Decay \(\lambda_t\) is drawn iid from \(F(\lambda)\), where the support of \(F\) includes 0, and observed by both players (but not yet experienced).

For ease of exposition, we restrict attention to the quadratic-loss quasilinear utility form introduced in Equation (1), although the results generalize readily to more general forms. It is also easier to deal with stochastic decay in discrete time, thus we now allow time to progress discretely, \(t = 1, 2, 3, \ldots\), without bound.

Proposition 4 shows that stochastic decay alone has minimal impact on equilibrium behavior. The equilibrium path is everywhere statically efficient yet dynamically inefficient in the aggregate. In every period, whenever decay is non-zero, a proposal is made that is accepted by the Voter, and the new policy either moves closer to the Proposer’s ideal point or returns it there. The only difference caused by stochastic decay is that the equilibrium path is now stuttered: moving in fits and starts of unequal size, following directly the irregular arrival of decay.

**Proposition 4** In the unique subgame perfect equilibrium a proposal \((x^*_t, 0)\) is offered in each period \(t\) and accepted, where:

\[
x^*_t = \min \left(\sqrt{x_{t-1}^2 + \frac{\lambda_{t-1}}{\alpha_V} \pi}, \pi\right)
\]
The equilibrium with stochastic decay has a very similar flavor to the deterministic case. Whenever decay appears it provides leverage to the Proposer to extract policy concessions from the Voter. The intermittent arrival of decay, however, means the acquisition of policy leverage is also intermittent and the time before policy arrives at the absorption point at the Proposer’s ideal point is also stochastic.

The trajectory of policy is determined, therefore, by $F(\lambda)$, the distribution of possible decay values. As the policy trajectory is determined by realized decay, this distribution can be smooth, continuous or discrete, without upsetting the equilibrium. For a distribution with atom at 0, the policy trajectory would consist of frequent inaction, interspersed with non-trivial movements in policy whenever non-zero decay appears. On the other hand, for a distribution with no atom at 0 and concentrated around a positive level, policy activity occurs in every period with a flow that approximates a regular rate.

5.1 Stochastic Decay and Temporal Expertise

As decay represents exogenously changing circumstances and the fit of policy to those circumstances, it is not to be expected that all policymakers possess the same information about decay. If decay’s arrival is unpredictable, policymakers need some mechanism to learn both if decay has appeared, and if so, how severe it is for the performance of the status quo policy. Such information is complex and multifaceted, representing the aggregated experiences of citizens and agents of the state. As such, it is a valuable commodity whose acquisition is costly. Some actors in government, then, are likely to have better tools and resources available for acquiring and analyzing this information than others.

To model this sort of expertise formally, we allow decay to be stochastic, as in the previous section, and we further amend Step 1 of the game to allow for information asymmetry.

Step 1a*. Decay $\lambda_t$ is drawn iid from $F(\lambda)$ and observed only by the Voter (but not yet experienced).

Note that in Step 4 of the game payoffs are realized and at this point the Proposer can infer that period’s decay. However, by the time the Proposer catches up on period $t$, time has ticked over and new decay has appeared, immediately restoring the Voter’s information advantage. In this way, the Voter’s expertise is short-lived but naturally recurring.
We characterize the equilibrium properties for this case in Proposition 5. The most notable of these properties is that decay is not immediately removed in each period and, consequently, that the equilibrium is statically as well as dynamically inefficient.

**Proposition 5** The unique subgame perfect equilibrium exhibits the following properties:

(i) For any period \( t \) status quo, \( (x_t,q_t) \), where \( x_t < \pi \), the policy proposal is rejected with positive probability.

(ii) The probability a proposal is rejected is strictly decreasing in \( |q_t| \).

(iii) For any realized set of decay shocks \( \lambda_1, \lambda_2, \ldots, \lambda_t \), the Voter’s utility in equilibrium is weakly greater than it would have been in the complete-information game with the same set of shocks.

(iv) With probability one, there is a \( t^* \) at which the proposal is \((\pi, 0)\) and it is accepted.

(v) For all \( t > t^* \), the policy proposal is \((\pi, 0)\) and it is accepted with probability one.

The Voter’s informational advantage means that the Proposer does not know the policies she is willing to accept. The only way to ensure an offer is accepted is to return policy to the efficient frontier without any policy concessions (as decay can be infinitesimally small or even zero). This is obviously unappealing to the Proposer as the logic recurs and policy would forever remain at the Voter’s ideal point. As a result, the Proposer trades-off the possibility of having his offer rejected against the benefit of policy concessions when it is accepted. Consequently, in equilibrium, the policy path now involves periods where offers are rejected and decay persists.

At the same time, the policy proposal is never so demanding that it is always rejected. The divide between accepted and rejected offers turns on the size of decay. In each period a critical threshold exists such that the Voter accepts if decay is greater than this level and rejects if it is below. This decision rule leads to the interesting property—part (ii) of the proposition—that decay is most likely to appear and accumulate when it arrives only slowly. In contrast, large bursts of decay are more likely to engender immediate action and policy change. Applied at large, this property predicts that a sudden crisis will engender an immediate policy response, whereas a slowly boiling crisis—like the proverbial frog in a pot
of water—is left to fester by policymakers until the maximal point is reached whereby action is unavoidable.

The emergence of decay in equilibrium is an inefficiency, and a static one at that. Moreover, it is a pure inefficiency in the sense that policymakers could remove it without cost if they wished (which contrasts with the inefficiency caused by proposal costs). Notably, however, the cost of this inefficiency is borne entirely by the Proposer. In fact, the Voter is strictly better off on this equilibrium path than she is when information is symmetric and policy remains on the efficient frontier. Any offer the Voter accepts is at least as good as when information is symmetric, and in most cases it is strictly better. Furthermore, because the Proposer’s utility function is (quasi-)concave over the ideological dimension, the uncertainty over acceptance causes him to play it safe and make proposals that demand smaller ideological concessions so as to reduce the risk of a rejection.

Nevertheless, the benefit to the Voter from her informational advantage lasts only a limited amount of time. Eventually the policy path reaches the Proposer’s ideal point, and when it does the policymakers share a common interest in removing decay and returning to the Proposer’s ideal. When this point is reached, the Voter accepts the offer for any level of decay and her informational advantage delivers no benefit. The appearance of decay in equilibrium is, thus, also a short-lived phenomenon. It is possible only in the interim period when ideological movement is possible and before it reaches its final stable point (which, again, contrasts with the equilibrium decay caused by proposal costs).

6 Concluding Discussion

An emerging and influential theme in American and comparative politics is that time matters. From Carpenter (2001)’s work on the history of the United States bureaucracy to Ertman (1997)’s account of state building in early modern Europe, the central lesson that has emerged is that by studying politics at single points in time–snapshots, so to speak–leads to not just an incomplete view of politics but a highly distorted view. (See Pierson (2004) for an excellent overview and theoretical synthesis).

Our paper fits tightly within this paradigm. Behavior within our model, to an outside observer, would appear to violate the logic of legislative gridlock and, indeed, the principles
of spatial voting. Our model provides the complete, dynamic logic behind such behavior. The model explains why a Voter, with policy at his ideal outcome today, will nevertheless vote for a policy tomorrow that is further from his ideal point. Without an understanding of the effects of time—that policy will have decayed in effectiveness—this behavior will not have been understood. Even more to the point, our result on the dynamic inefficiency of the policy path, by definition, would not be evident in a one-shot, or static, view of politics.

In taking time seriously, the challenge of explaining policy takes on a different and broader perspective. As articulated by Hall (1993), a theory must now explain what causes change as well as what causes periods of stability. In our benchmark model, the irrepressible arrival of decay provides the fuel for policy change, and left unencumbered, policy change will be equally irrepressible. Change is frequent—in fact, continuous in the extreme—and each change is incremental. On the other hand, if one policymaker possesses expertise, or if legislative costs are significant, then the wheels of change slow down. Decay is not always removed instantaneously and policy negotiations often break down. This generates a policy path that is intermittent, with more substantial change occurring periodically.

These different patterns of policymaking correspond to different perspectives on U.S. policymaking. A classic theme on political decision making is that change should be incremental (Lindblom, 1959). This corresponds to an environment where frictions are minimal. The logic driving incrementalism in our setting, however, could not be more different from the uncertainty and cognitive limitations that motivated Lindblom.

An alternative, more recent view, on dynamic policymaking is the idea of punctuated equilibrium, introduced independently in Baumgartner and Jones (1993) and Hall (1993). A pattern of this sort—periods of lengthy stability punctuated by substantial change—corresponds, in our setting, to an environment where expertise is substantial.\textsuperscript{16} Once again, however, the driving force behind this pattern is very different in our model. Indeed, although policy experiences periods of inaction, the environment as a whole is not stable as decay builds up in plain sight. Policy is decaying and everyone can agree that ‘something must be done’ yet they cannot agree on what that should be done, and the inefficiency of decay festers and grows.

\textsuperscript{16}This pattern is magnified if the distribution of decay is skewed with most mass on low levels of decay and the small possibility of large decay.
Our work complements these previous contributions in using the tools of formal theory to study how policymaking overcomes the challenge of change. Ours is the first to formalize the concept of decay, although a recent paper by Callander and Krehbiel (2014) develops a related model of drift in line with the distributional examples in Hacker and Pierson (2010). The concepts of drift and decay are formally distinct and generate distinct sets of implications for policymaking. Namely, with policy drift, legislative gridlock does not break down; indeed, gridlock is only strengthened as the prospect of drift heightens the costs of legislative compromise. Callander and Krehbiel (2014) show how these inefficiencies can be overcome, at least in part, by the strategic delegation of authority. We also are distinguished in developing a fully dynamic model rather than a simple two-stage game. This allows the complete incentives of policymakers to apply to behavior, and to understand how change—via decay—can accumulate over time. Our contribution also serves to address a shortcoming of formal models of legislative politics that all too often abstract away from forward looking behavior and the effects of time on policy.\footnote{As articulated by Moe (2013) (p. 1175-1176): “Another troubling constraint built into the normal framing, an exceedingly consequential one, is that actors are not forward looking. ... But even though this forward-looking logic could not be more basic to rational behavior and is often discussed, it is simply omitted from formal models of delegation and control. Among those doing the modeling, this omission is not regarded as a problem, or even an issue worth pointing out. It is normal. In essence, then, existing models assume that their actors behave in rather stupid ways. They are fixated on the present, as though politics is never going to change.”}

This dynamic perspective also allows for the emergence of a novel conception of bureaucratic expertise, a conception that is not possible in a static model. The standard notion of policy expertise, as in Gilligan and Krehbiel (1987), involves knowledge of the mapping between policies and outcomes. That is, which policy tools work best to achieve a desired outcome? This kind of expertise is static, as once a policy is implemented the knowledge becomes public thereafter.

Our conception of expertise is more appropriately thought of as an advantage in the speed of learning. This advantage is transient, as both players will eventually learn the facts on the ground, but it is continually renewed as the world continues to change and new developments continue to arise.

In particular, we expect that the executive branch—which is charged with implementing policy—is better able to identify decay when it occurs. The executive has direct access to
a vast hierarchy of agents (bureaucrats) on the ground who are involved in implementing policies and are the first to encounter new challenges to implementation that arise over time. The legislature, on the other hand, is external to the bureaucracy and can query its agents only through slow-moving formal processes such as oversight hearings and subpoenas. Thus, it is reasonable to expect that when policy bargaining takes place, it takes place in the presence of asymmetric information, with the executive possessing more up-to-date information about the deleterious effects of decay than does the legislature.

The legislative bargaining within the model is notable both for its efficiency and its inefficiency. Legislators possess an impressive ability to strike deals that remove decay and restore policy to the efficient frontier. Nevertheless, this ability is imperfect, and the failures, both statically and dynamically, imply a failure of institutional design. How to solve these problems through better design is an obvious question to pursue.

A first solution is to leave the institutions in place but change the policies that are used. If policies could self-correct for decay then legislative action would not be necessary to remove it and temporal expertise would be moot. Self-correcting policies may sound fantastical, and too good to be true, but they are already a reality. The most prominent example is the indexing of Social Security to inflation. Price changes erode purchasing power, and to avoid this type of decay in retirement benefits, Congress passed legislation that self-corrects for inflation. This type of self-correcting policy, however, is best thought of as the exception that proves the rule. Measuring the purchasing power of a dollar is perhaps the most definable and measurable type of decay possible, and even then the approach is not without controversy (see the debate over using chained versus unchained CPI measures of inflation). In most cases the type of change is unknown and unknowable, and how it impacts policy difficult to predict. Writing legislation that accounts for the innumerable possibilities is nearly impossible, and the possibility of self-correcting policies is of limited practical use.18

Another potential solution lies in the bureaucracy. If policies cannot self-correct themselves, perhaps bureaucrats can do it instead. By empowering bureaucrats to ‘fix’ decayed policies, policymakers can avoid the inefficiencies, and the headache, of decay altogether. However, avoiding the headache of decay also removes the benefits of decay, and to the possessor of agenda control, an empowered bureaucracy only serves to suppress the leverage

18This logic resonates with the enormous literature in economics on the logic behind incomplete contracts.
that would otherwise accrue. This logic leads to a surprising, yet clear, connection to the long-standing puzzle in bureaucratic politics on the design of agencies.

It is a notorious fact in U.S. policymaking that the design of the bureaucracy is labyrinthine and, in many respects, inefficient. Moe (1989, p. 267) famously argues that the inefficiencies are intentional.

“American public bureaucracy is not designed to be effective. The bureaucracy arises out of politics, and its design reflects the interests, strategies, and compromises of those who exercise political power.”

Moe refers to this inefficiency as “policy insulation,” an effort to cement temporary political power into a lasting bureaucratic structure. Our model of decay provides an entirely different logic for the complexity. Just as formal rules and procedures make it hard for agencies to adapt to new strategic direction at the top, they also make it hard to adapt to exogenous technological or social changes. By strangling the ability of agencies to respond to decay, the agenda setter therefore locks-in his power over legislative action. In contrast to Moe’s rationale, inefficiency is the objective of bureaucratic design in our theory and not merely an unavoidable by-product.

At the heart of bureaucratic design is the same question that animates much of American politics: How do the different branches of government, in particular the executive and legislative branches, stand in relative power over policymaking. The principal insight we offer here is that the answer to this question turns on the allocation of agenda control with even greater force than previously thought. To the extent that this is the Congress in U.S. policymaking, our model has provided a new foundation for Congressional dominance over the executive branch.

References


19 This logic is consistent with the findings in Howell and Lewis (2002) and Lewis (2004) that agencies designed by Congress are more long-lasting than those designed within the executive branch.


Mayhew, David R., Divided we govern, Yale University, 1991.


Appendices

A Proofs

A.1 Proof of Proposition 1 (Myopic Equilibrium)

For a policy starting at an arbitrary point \((x_t, q_t)\), the myopic voter’s indifference condition between allowing an infinitesimal amount of decay and accepting an infinitesimal movement \((dx, dq)\) is given by:

\[
 u_{V,t}(x_t, q_t - \lambda dt) = u_{V,t}(x_t + dx, q_t + dq)
\]

Applying a first-order Taylor expansion around the point \((x_t, q_t)\) to each side, we get:

\[
 u_{V,t}(x_t, q_t) - \frac{\partial u_{V,t}}{\partial q} \lambda dt = u_{V,t}(x_t, q_t) + \frac{\partial u_{V,t}}{\partial x} dx + \frac{\partial u_{V,t}}{\partial q} dq
\]

\[
 dx = - \left( \frac{\partial u_{V,t}}{\partial q} \frac{\partial u_{V,t}}{\partial x} \right) (\lambda + dq) dt
\]

Here, \(\frac{dx}{dt}\) is the rate of change in the equilibrium policy with respect to time. A few properties of the equilibrium path are evident from equation 8. First, if \(\frac{dx}{dt}\) is to be positive - that is, if policy is to move in the direction of the proposer - then \(\frac{dq}{dt} \geq -\lambda\). In other words, the proposer can only extract policy concessions from the voter by offering to eliminate some or all of the decay that would be occurring in the absence of an agreement. Second, the rate at which policy moves in the direction of the proposer is increasing in \(\frac{dq}{dt}\). For an initial policy that is on the efficient frontier, \(q\) cannot increase, implying that the proposer would like to set \(\frac{dq}{dt}\) to zero, i.e., remain on the efficient frontier. In this case, (8) simplifies to:

\[
 \frac{dx}{dt} = -\lambda \left( \frac{\partial u_{V,t}}{\partial q} \frac{\partial u_{V,t}}{\partial x} \right)
\]

Indicating that the rate at which the proposer extracts concessions increases when either
the rate of decay or the voter’s marginal rate of substitution between efficiency and policy increases. To find an expression for the equilibrium policy path, we rewrite the above as:

\[
\frac{\partial u_{V,t}}{\partial x} \frac{dx}{dt} + \frac{\partial u_{V,t}}{\partial q} \frac{dq}{dt} = -\lambda \frac{\partial u_{V,t}}{\partial q}
\]

The left-hand side of the above is the total derivative of \(u_{V,t}\) with respect to \(t\). Integrating both sides from 0 to \(t\) with respect to \(t\), we get:

\[
\int_0^t \frac{du_{V,t}}{dt'} dt' = u_{V,t}(x(t), q(t)) - u_{V,t}(x(0), q(0)) = u_{V,t}(x(t), q(t)) = -\lambda \int_0^t \frac{\partial u_{V,t}}{\partial q} dt'
\]

Again, under the proposer’s optimal path, \(q(t) = 0 \ \forall t\). By the assumptions that \(u_{V,t}\) is quasiconcave and maximized at \(x = 0\), \(u_{V,t,0}(x)\) - i.e., the slice of \(u_{V,t}\) holding \(q\) fixed at zero - is invertible when \(x > 0\). Because the proposer’s utility is maximized at \(x = \pi\), she has no further incentive to offer rightward movements in policy once that point has been reached, and the voter will always accept an offer to eliminate decay but remain at the current policy position. Putting these two points together, we get the full equilibrium path, given by:

\[
x(t) = \min \left[ u_{V,t,0}^{-1} \left( -\lambda \int_0^t \frac{\partial u_{V,t}}{\partial q} dt' \right), \pi \right]
\]  

(A.2) Proof of Proposition 2 (Far-sighted Equilibrium)

The continuity of the equilibrium path is a direct consequence of subgame perfection. Suppose the Voter considers an immediate jump from \((x_0, 0)\) to \((x_1, 0)\), where \(x_1 > x_0\). The Voter must anticipate that the path following \(x_1\) will be identical, whether that point is arrived at via this immediate jump or by any other path. From this it follows immediately that the Voter strictly prefers any path to \(x_1\) along which his utility changes continuously - and in particular, the path involving allowing decay to take hold until the point of instantaneous indifference with \(x_1\) - to the path with a jump, where his utility drops discontinuously. He therefore rejects the offer of \((x_1, 0)\). So long as the equilibrium policy remains on the efficient frontier, its motion must be smooth.

Decay is ruled out in equilibrium because it implies a contradiction. Suppose that the
Proposer prefers some path from \((x_0, 0)\) to \((x_1, 0)\) which involves allowing decay to take hold to the best (smooth) path between those points, \(x_s(t)\), which remains on the efficient frontier and would be acceptable to the Voter. For this to be possible, it is necessary that the path \(x_s(t)\) takes longer to arrive at \(x_1\) than does the path involving decay - any path which arrived in shorter time while remaining on the frontier would be strictly preferred by the Proposer. But if this is the case, then the Voter strictly prefers \(x_s(t)\) to the path involving decay. This can be seen by comparing the Voter’s utility from the two options:

\[
\begin{align*}
  u_V(x_s(t)) &= \int_0^{t^*} u_{V,t}(x_s(t), 0)e^{-\delta t}dt + \int_{t^*}^{\infty} u_{V,t}(x^*(t - (t^* - t^{**})), 0)e^{-\delta t}dt \\
  u_V(decay) &= \int_0^{t^{**}} u_{V,t}(0, -\lambda t)e^{-\delta t}dt + \int_{t^{**}}^{\infty} u_{V,t}(x^*(t), 0)e^{-\delta t}dt
\end{align*}
\]

Where \(t^*\) is the time elapsed between \(x_0\) and \(x_1\) when following \(x_s(t)\); \(t^{**}\) is the time elapsed when allowing decay to take hold (e.g., the time until arrival at the Voter’s indifference point with \(x_1\)); and \(x^*(t)\) is the equilibrium path played following the arrival at \(x_1\) at time \(t^{**}\). If \(t^{**} < t^*\), it is easily verified that the Voter strictly prefers the offer \(x_s(t)\) to his outside option of holding out, which contradicts the assumption that \(x_s(t)\) is the minimum offer that the Voter would accept.

A.3 Proof of Corollary 2 (Comparative statics on rate of movement)

It suffices to consider the Voter’s forward-looking utility beginning from some point \((x_0, 0)\) when she considers holding out until the boundary point, as this utility level provides a lower bound for the utility she experiences under the equilibrium offers. This utility is:

\[
u_V = \int_0^{t^{**}} u_{V,t}(x_0, -\lambda t)e^{-\delta t}dt\]
\( \frac{\partial^2}{\partial q^2} u_{V,t} < 0 \) (accelerating costs of decay away from the frontier) implies that this integral is more negative than in the case of constant marginal cost \( \left( \frac{\partial^2}{\partial q^2} u_{V,t} = 0 \right) \). The forward-looking voter is thus willing to accept a faster rate of movement than that given by the myopic path, which is defined by the marginal rate of substitution at the efficient frontier. Similarly, \( \frac{\partial^2}{\partial q^2} u_{V,t} > 0 \) implies that the Voter’s reservation utility is higher.

### A.4 Proof of Proposition 3 (Equivalence with Myopic Path)

At time zero, a forward-looking voter who anticipated following the path defined by equation 2 would expect utility given by:

\[
\begin{align*}
    u_v &= \int_0^\infty e^{-\delta t} u_{V,t} \left( \min \left[ u_{V,t,0}^{-1} \left( -\lambda \int_0^t \frac{\partial u_{V,t}}{\partial q} dt' \right), \pi \right], 0 \right) dt \\
    &= \int_0^{t^*} e^{-\delta t} u_{V,t} \left( u_{V,t,0}^{-1} \left( -\lambda \int_0^t \frac{\partial u_{V,t}}{\partial q} dt' \right), 0 \right) dt + \int_{t^*}^\infty e^{-\delta t} u_{V,t} (\pi, 0) dt \\
    &= \int_0^{t^*} e^{-\delta t} \left( -\lambda \int_0^t \frac{\partial u_{V,t}}{\partial q} dt' \right) dt + \int_{t^*}^\infty e^{-\delta t} u_{V,t} (\pi, 0) dt
\end{align*}
\]

where \( t^* \) is the time it takes to reach the absorbing state at the proposer’s ideal point, \( \pi \), and defined implicitly by the equation \( \pi = u_{V,t,0}^{-1} \left( -\lambda \int_0^{t^*} \frac{\partial u_{V,t}}{\partial q} dt' \right) \).

On this path the voter can threaten the proposer with rejection. Doing so leaves the voter indifferent – by the equilibrium condition – yet it lowers the proposer’s utility who now experiences a less attractive policy location and inefficiency. The worst threat the voter can hold over the proposer, then, is to reject all proposals. The question of interest is whether this allows the voter to extract a better offer from the proposer. Unfortunately, for the voter, the answer is no.

So suppose the voter rejects all offers and policy decays. At all points the voter is indifferent between this path and what she would have experienced on the myopic path and the proposer is significantly worse off. This logic stops, however, once a point \( (0, -q_t^*) \) is reached at which she is indifferent between that point and the proposer’s ideal point \( (\pi, 0) \). Below this point the voter is strictly worse off rejecting offers than moving to the myopic
path. This is because the proposer’s ideal point is an absorption point on the myopic path. On the myopic path policy gets no worse for the voter once \((\pi, 0)\) is reached, whereas by rejecting offers decay continues and the voter’s utility continues to decrease without bound.

It is at the point \((0, -q_1^*)\) that subgame perfection binds. Once this point is reached, the voter accepts the offer of the proposer to move to \((\pi, 0)\) and accepts every offer thereafter that keeps policy at the proposer’s ideal. So suppose the voter engages in this worst-case threat, rejecting all offers until point \((0, -q_1^*)\) and accepting all offers thereafter. Her time 0 utility is then given by:

\[
\begin{aligned}
u_V &= \int_0^{t^{**}} e^{-\delta t} u_{V,t}(0, -\lambda t) \, dt + \int_{t^{**}}^{\infty} e^{-\delta t} u_{V,t}(\pi, 0) \, dt \\
&= \int_0^{t^{**}} e^{-\delta t} u_{V,t}(0, -\lambda t) \, dt + \int_{t^{**}}^{\infty} e^{-\delta t} u_{V,t}(\pi, 0) \, dt \\
&= (13)
\end{aligned}
\]

where \(t^{**}\) is the amount of time it takes for decay to reach the voter’s point of indifference, and defined implicitly by the equation \(u_{V,t}(\pi, 0) = u_{V,t}(0, -\lambda t^{**})\). In general, the utilities given by equations 12 and 13 may differ. If the utility in (13) exceeds that in (12), the forward-looking voter is not willing to accept the myopic path; the proposer will still be able to construct an offer that forestalls decay, but the deal will be better (from the voter’s perspective) than that received by the myopic voter.

Here we note that, if the voter’s utility is quasilinear in the quality dimension, that is, \(u_{V,t}(x, q) = f(x) + q\) for some function \(f\), equations 12 and 13 simplify dramatically, yielding a striking equivalence. In this case, \(\frac{\partial u_{V,t}}{\partial q} = 1\), which implies:

\[
-\lambda \int_0^t \frac{\partial u_{V,t}}{\partial q} \, dt' = -\lambda t
\]

Quasilinearity also yields:

\[
u_{V,t}(0, -\lambda t) = -\lambda t + f(0) = -\lambda t
\]

Where the second equality follows from the normalization \(u_{V,t}(0, 0) = 0\). Hence, in the quasilinear case the integrands of equations 12 and 13 are identical. The only question determining how the voter compares the two is how \(t^*\) compares to \(t^{**}\). But \(t^{**}\) solves the indifference condition:

40
\[ f(\pi) = -\lambda t^{**} \]

implying that

\[ t^{**} = -\frac{f(\pi)}{\lambda} \]

Whereas \( t^{*} \) solves the boundary condition

\[ \pi = u_{V,t,0}^{-1}(-\lambda t^{*}) \]

implying that

\[ t^{*} = -\frac{f(\pi)}{\lambda} = t^{**} \]

Hence, with quasilinear utility the forward-looking voter is exactly indifferent between engaging in the worst-case threat, and following the myopic path. From the perspective of time 0, the voter is exactly indifferent between following the path given in equation 3 and holding out until \( t^{*} \), then moving all at once to the proposer’s ideal policy \( \pi \). As the proposer clearly strictly prefers the first option, the path in equation 3 is robust to this deviation.

The remainder of the proof is to then consider deviations at all other points on the equilibrium path. These deviations take a similar form to the worst-case deviation at time 0 just analyzed. Suppose play is at time \( \tau > 0 \) and that players have been following the path in (3) for some time. The voter’s forward-looking utility from continuing on this path is:

\[
\int_{\tau}^{t^{*}} e^{-\delta(t-\tau)} f(f^{-1}(-\lambda t)) \, dt = -\int_{\tau}^{t^{*}} e^{-\delta(t-\tau)} \lambda t \, dt
\]  

(14)

Starting from \( x_{\tau} = f^{-1}(-\lambda \tau) \), the voter’s threat to hold out yields utility of:

\[
\int_{\tau}^{\tau+\tilde{t}} e^{-\delta(t-\tau)} (f(x_{\tau}) - \lambda(t-\tau)) \, dt = -\int_{\tau}^{\tau+\tilde{t}} e^{-\delta(t-\tau)} \lambda t \, dt
\]  

(15)

\( \tilde{t} \) here is the time it takes to reach the voter’s indifference point with the jump to \( \pi \), i.e.

\[-\lambda \tilde{t} + f(x_{\tau}) = f(\pi) \]  

(16)
Which implies \( \tilde{t} = -\frac{f(\pi) + \lambda \tau}{\lambda} = t^* - \tau \). Plugging in, we see that the utility in (14) is exactly equal to the utility in (15). Hence, at all times \( \tau \geq 0 \), the voter’s participation constraint is exactly satisfied. The myopic voter path is thus an equilibrium even if voters are forward-looking.

A.5 Proof of Remark 1 (Proposal costs)

In the variant of the model with positive proposal cost, we conjecture an equilibrium path consisting of a finite number \( k \) of proposals occurring at times \( \tau_1, \tau_2, \ldots, \tau_k \) and associated with proposals \((x_1, 0), (x_2, 0), \ldots, (x_k, 0)\), before the game reaches the end state at \( x_{k+1} = \pi \).\(^{20}\)

Consider the last \((k^{th})\) such proposal. The choice the Proposer faces is between proposing (and paying the cost) now, versus waiting an additional instant, getting a slightly better deal from the voter, but experiencing additional decay in the interim. Proposing now (at time \( \tau_k \)) yields utility:

\[
\begin{align*}
    u_P &= -c + \int_{\tau_k}^{t^*} u_{P,t}(x_k, -\lambda(t - \tau_k))e^{-\delta(t - \tau_k)}dt + V_{\pi} \\
\end{align*}
\]

whereas waiting an amount of time \( \Delta \tau \) would yield:

\[
\begin{align*}
    u_P &= -e^{-\delta \Delta \tau} c + \int_{\tau_k}^{\tau_k + \Delta \tau} u_{P,t}(x_{k-1}, -\lambda(t - \tau_{k-1}))e^{-\delta(t - \tau_k)}dt \\
    &\quad + \int_{\tau_k + \Delta \tau}^{t^*} u_{P,t}(x_k', -\lambda(t - \tau_k))e^{-\delta(t - \tau_k)}dt + V_{\pi}
\end{align*}
\]

Where \( x_k' \) is the voter’s new point of indifference on the frontier after an additional amount \( \lambda \Delta \tau \) of decay takes hold. Rearranging, we get that proposing now is optimal only if \( c \) is less than some upper bound:

\(^{20}\)Note that with proposal costs, the single point \((\pi, 0)\) is no longer an absorbing state. Instead, the players cycle through a vertical line below \((\pi, 0)\), allowing some decay to take hold before periodically agreeing to return to the frontier but remain at \( \pi \) on the \( x \)-axis.
As we let $\Delta \tau$ go to zero, $x'_k \to x_k$, and hence both the numerator and the denominator of the ratio defining the upper bound on $c$ converge to zero. Application of L’Hopital’s rule and the Leibniz integral rule gives:

$$\lim_{\Delta \tau \to 0} \frac{A + B}{e^{-\delta \Delta \tau} - 1} = \frac{u_{P,t}(x_k,0) - u_{P,t}(x_{k-1},-\lambda(t-k_{-1}))}{\delta}$$

(17)

The numerator of equation 17 is just the difference in instantaneous utility (for the proposer) between the current point $(x_{k-1},-\lambda(t-k_{-1}))$ and the proposed point $(x_k,0)$. Identical logic prevails as we recurse backwards through the remainder of the path, replacing $V_\pi$ in the above with $V_{x_k}$.

### A.6 Proof of Remark 2 (Variable Efficient Frontier)

This section uses the quasilinear functional form with quadratic-loss utility over the $x$ dimension; generalization to other quasilinear forms is straightforward. The voter considers holding out for an amount of time $\Delta \tau$, compared to accepting an offer to move to $(\hat{x}, \hat{q})$ immediately. Note that for small delay times $\Delta \tau$, the Proposer’s subsequent offer will still be $(\hat{x}, \hat{q})$. In either case, once $(\hat{x}, \hat{q})$ is reached, the game will proceed in exactly the same fashion regardless of the path taken to arrive there, and hence the Voter’s choice is between experiencing the equilibrium path beginning at $(\hat{x}, \hat{q})$ now, or pushing it into the future by an amount of time $\Delta \tau$. The Voter’s forward looking utility for the path involving delay is:

$$u_V(wait) = \int_{0}^{\Delta \tau} (-\lambda t) dt + e^{-\delta \Delta \tau} u_V(jump)$$
Some algebra gives that for accepting now to be optimal, it must be the case that:

\[ u_V(\text{jump}) > \frac{\lambda}{\delta} \left( \frac{\Delta \tau}{e^{\delta \Delta \tau} - 1} - \frac{1}{\delta} \right) \]  

(18)

The path after the “jump” to \((\hat{x}, \hat{q})\) has three distinct phases. In the first, policy slides down from \((\hat{x}, \hat{q})\) to \((\hat{x}, 0)\) as decay takes hold, but with \(q > 0\) the Proposer lacks leverage to move policy any further right.\(^{21}\) In the second phase, beginning from \((\hat{x}, 0)\) the game is identical to that studied in the baseline case, and hence the trajectory is the same as that described in Equation (3) for \(x > \hat{x}\). Finally, in the third phase the policy remains at \((\pi, 0)\) forever. The voter’s utility from each of these phases is:

\[
\begin{align*}
    u_V(\text{Phase 1}) &= \int_0^{\hat{q}/\lambda} e^{-\delta t}(-\alpha V \hat{x}^2 + \hat{q} - \lambda t)dt \\
    u_V(\text{Phase 2}) &= \int_{\hat{q}/\lambda}^{\hat{q}/\lambda + t^*(\hat{x})} e^{-\delta t}(-\alpha V x^*(t + t(\hat{x}) - \hat{q}/\lambda)^2)dt \\
    u_V(\text{Phase 3}) &= \int_{\hat{q}/\lambda + t^*(\hat{x})}^{\infty} e^{-\delta t}(-\alpha V \pi^2)dt
\end{align*}
\]

Where \(t^*\) is as defined in Section 3, \(t(\hat{x})\) is the time it would have taken to reach \(\hat{x}\) under the equilibrium path described in Equation (3), and \(x^*\) is the location of that same path at a given time.

For movement to \((\hat{x}, \hat{q})\) to be statically efficient, it must be the case that \(\hat{q} \geq \alpha V \hat{x}^2\). Consider the case when this condition is exactly satisfied; then \(t(\hat{x}) = \alpha V \hat{x}^2/\lambda = \hat{q}/\lambda\), and the sum of the above three utilities simplifies to:

\[ u_V(\text{jump}) = \frac{\lambda}{\delta^2} (e^{-\delta t^*} - 1) \]

For \(\pi > 0\), \(t^* > 0\), and hence for any \(\delta \in (0, \infty)\), i.e. a Voter who discounts the future but is not perfectly myopic, the above is strictly less than zero. Note that the limit of the right hand side of the inequality in (18) as \(\Delta \tau \to 0\) is exactly 0, and hence there is some

\(^{21}\)Note that we are presuming here a Proposer who prefers \((\pi, 0)\) to \((\hat{x}, \hat{q})\); if not, \((\hat{x}, \hat{q})\) would be the absorbing state.
positive $\Delta \tau$ which the voter prefers to wait rather than jump to $(\hat{x}, \hat{q})$ immediately. Again by the strictness of the inequality, this preference continues to hold even if $\hat{q} > \alpha_V \hat{x}^2$.

### A.7 Proof of Proposition 4 (Stochastic Equilibrium Path)

Letting $\zeta = 1/(1 + \delta)$, instantaneous and total utility in discrete time are given by:

\[
\begin{align*}
u_{V,t} &= -\alpha_V x_t^2 + q_t \\
u_V &= \sum_{t=0}^{\infty} \zeta^t u_{V,t}
\end{align*}
\]

Again, subgame perfection implies a boundary below which the game effectively ends. This boundary is defined by:

\[
-\alpha_V x_0^2 + q_0 + \sum_{t=1}^{\infty} \zeta^t \alpha_V \pi^2 \leq \sum_{t=0}^{\infty} \zeta^t \alpha_V \pi^2 \\
\Rightarrow |q_0| \geq \alpha_V (\pi^2 - x_0^2)
\]

From any given starting point $(x, q)$, define:

\[
\begin{align*}
\lambda(x, q) &\equiv \alpha_V (\pi^2 - x^2) - |q| \\
p(x, q) &\equiv \mathbb{P}(\lambda_t \leq \lambda(x, q)) = F(\lambda(x, q)) \\
V(x, q) &\equiv E_{\lambda}[u_V(x, q)|S^*_v, S^*_p] \\
V_\pi &\equiv \sum_{i=0}^{\infty} -\alpha_V \pi^2
\end{align*}
\]

Where $(S^*_v, S^*_p)$ are equilibrium strategies of the voter and the proposer respectively. $\lambda$ is the maximum amount of decay that could occur without moving the game outside of the boundary and effectively ending it, and $p(x, q)$ is then the probability that the game continues next period. Then, the proposer’s optimal proposal at any point $(x_0, q_0)$ (where $q_0$ here is taken to be the location of the status quo after that period’s decay has been realized)
is given by:

\[
x^*(x_0, q_0) = \max_x \quad \text{s.t.} \quad -\alpha V x^2 + \zeta V(x, 0) = -\alpha V x_0^2 + q_0 + \zeta V(x_0, q_0)
\]

\[
\Rightarrow \quad \zeta [V(x^*(x_0, q_0), 0) - V(x_0, q_0)] = \alpha V (x^*(x_0, q_0)^2 - x_0^2) + q_0
\]

\[
\Rightarrow \quad x^*(x_0, q_0) = \sqrt{x_0^2 - \frac{q_0}{\alpha V}} + \zeta [V(x^*(x_0, q_0), 0) - V(x_0, q_0)]
\]

Conjecture \(x^*(x_0, q_0) = \sqrt{x_0^2 - \frac{q_0}{\alpha V}}\). We can show that this policy sets \(V(x^*(x_0, q_0), 0) - V(x_0, q_0) = 0\), and hence is in fact a solution to the proposer’s maximization problem. Note that:

\[
V(x^*(x_0, q_0), 0) = p(x^*(x_0, q_0), 0) E \left[ \alpha V x^*(x_0, q_0, -\lambda)^2 + \zeta V(x^*(x_0, q_0, -\lambda), 0) | \lambda < \bar{\lambda}(x^*(x_0, q_0), 0) \right]
\]

\[
+ (1 - p(x^*(x_0, q_0), 0)) V_\pi
\]

\[
V(x_0, q_0) = p(x_0, q_0) E \left[ \alpha V x^*(x_0, q_0 - \lambda)^2 + \zeta V(x^*(x_0, q_0), q_0 - \lambda) | \lambda < \bar{\lambda}(x_0, q_0) \right]
\]

\[
+ (1 - p(x_0, q_0)) V_\pi
\]

Now consider the components of the value functions above. First, the next-period payoffs:

\[
\alpha V x^*(x_0, q_0, -\lambda)^2 = \alpha V x^*(x_0, q_0)^2 + \lambda
\]

\[
= \alpha V x_0^2 - q_0 + \lambda
\]

\[
\alpha V x^*(x_0, q_0 - \lambda)^2 = \alpha V x_0^2 - q_0 + \lambda
\]

So, the terms inside the expectation operators are the same. To show that the expectations are equal, we need to show that the conditioning events are also the same:

\[
\bar{\lambda}(x^*(x_0, q_0), 0) = \alpha V (\pi^2 - x^*(x_0, q_0)^2) = \alpha V (\pi^2 - x_0^2) - |q_0|
\]

\[
\bar{\lambda}(x_0, q_0) = \alpha V (\pi^2 - x_0^2) - |q_0|
\]
Hence, \( x^*(x_0, q_0) = \sqrt{x_0^2 - \frac{q_0}{\alpha_V}} \) exactly satisfies the voter’s participation constraint for any \((x_0, q_0)\). The equilibrium to the game thus involves the proposer offering the sequence of offers:

\[
x_t^* = \min \left( \sqrt{x_{t-1}^2 + \frac{\lambda_t - 1}{\alpha_V}}, \pi \right)
\] (19)

And the voter accepting. Decay never persists for a full period along the equilibrium path.

A.8 Proof of Proposition 5 (Asymmetric Information)

The analysis of this case is very close to the case with perfect information. In fact, stages 3 and 4 (after the proposer makes an offer) are identical to the full-information case. This is because the proposer is uninformed at the time of the offer: the offer he makes cannot condition on the current or future periods of decay and thus cannot provide any information to the voter. Hence, the voter behaves in the same way, conditional on a particular offer, that he would in the complete-information game. In particular, the same boundary obtains which, if it is crossed at any point, effectively ends the game.

However, the second stage of the game is substantially different when the proposer is uninformed about the current level of decay. The proposer can no longer exactly satisfy the voter’s participation constraint:

\[
\alpha_V x^2 \leq \alpha_V x_0^2 - q_0 + \lambda_t + \zeta(V(x, 0) - V(x_0, q_0 - \lambda_t))
\] (20)

Because he does not know \( \lambda_t \). If the support of \( F(\lambda) \) includes zero, this property leads immediately to the conclusion that under asymmetric information, there must be offers rejected and decay allowed to persist in equilibrium (part (i) of the Proposition). The reason that rejections appear on the equilibrium path is that to guarantee that all voter types would accept, the proposer would need to offer:

\[
x^2 = x_0^2 - \frac{q_0 + \zeta(V(x, 0) - V(x_0, q_0))}{\alpha_V}
\]

But the path consisting of remaining at \((0,0)\) forever is the only solution to this equation, and therefore this strategy cannot be an equilibrium. The proposer must make an offer strictly
worse (for the voter) than this minimum, and hence for sufficiently small $\lambda$, the voter rejects the offer and decay persists.

Now, we examine again the voter’s participation constraint for an arbitrary type $\lambda_t$, and the offers it induces from the proposer. A few lemmas are useful here:

**Lemma 1** $V(\sqrt{x_0^2 - q_0/\alpha V}, 0) = V(x_0, q_0)$

**Proof.** Suppose the proposer follows the strategy $x = \sqrt{x_0^2 - q_0/\alpha V}$. Then:

$$V(x_0, q_0) = E_\lambda \left[ \max \left( -\alpha V \left( \sqrt{x_0^2 - q_0/\alpha V} \right)^2 + \zeta V \left( \sqrt{x_0^2 - q_0/\alpha V}, 0 \right), -\alpha x_0^2 + q_0 - \lambda + \zeta V(x_0, q_0 - \lambda) \right) \right]$$

$$V\left( \sqrt{x_0^2 - q_0/\alpha V}, 0 \right) = E_\lambda \left[ \max \left( -\alpha V \left( \sqrt{x_0^2 - q_0/\alpha V} \right)^2 + \zeta V \left( \sqrt{x_0^2 - q_0/\alpha V}, 0 \right), -\alpha x_0^2 + q_0 - \lambda + \zeta V\left( \sqrt{x_0^2 - q_0/\alpha V}, -\lambda \right) \right) \right]$$

Repeated application of the same logic to the terms $V\left( \sqrt{x_0^2 - q_0/\alpha V}, -\lambda \right)$ and $V(x_0, q_0 - \lambda)$ demonstrates the equivalence, since ultimately both chains terminate at $V_\pi$, the value of being at point $(\pi, 0)$ forever. ■

**Lemma 2** If the point $(x_t, q_t)$ is reached, then the proposer’s offer at $t + 1$, $x_{t+1}^P$, is at least $\sqrt{x_t^2 - q_t/\alpha V}$.

**Proof.** From the discussion above, the minimum non-dominated offer that the proposer could make sets $x^2 = x_0^2 - \frac{q_t + \zeta (V(x, 0) - V(x_0, q_0))}{\alpha V}$ Application of lemma 1 implies $x_{t+1}^P \geq \sqrt{x_t^2 - q_t/\alpha V}$. ■

**Lemma 3** For any type $\lambda_t$ that accepts an offer in equilibrium, $\zeta (V(x, 0) - V(x_0, q_t - \lambda_t)) \geq 0$.

**Proof.** Suppose not. Then $x_t < \sqrt{x_{t-1}^2 - q_t/\alpha V}$ for some $t$. ■

If the support of the density $F$ is continuous, then there exists a type $\tilde{\lambda}$ which is indifferent between accepting and rejecting. For this type, $V(x, 0) - V(x_0, q_0 - \lambda_t) = 0$. Given 20,

$$\tilde{\lambda} = \alpha_p (x^2 - x_0^2) + q_0$$

(21)
Given this cutoff value, the probability of an offer being accepted is \( p(x, x^0, q_0) = 1 - F(\tilde{\lambda}) \).

The proposer’s problem is to solve:

\[
\max_x -(1 - F(\tilde{\lambda}))(\alpha_P)(\pi^2 - x^2) + F(\tilde{\lambda})[-\alpha_P(\pi^2 - x_0^2) + q_0 - E[\lambda]]
\]  

(22)

The first order conditions on this problem yield the condition:

\[
x = F(\tilde{\lambda})(x - \pi) + xf(\tilde{\lambda})(\alpha_P(x^2 + 2\pi(x_0 - x)) + q_0 - E[\lambda])
\]  

(23)

This equilibrium condition does not admit a closed form solution, even for some simple densities like the uniform. However, concave utility implies that the proposer’s offer is more favorable to the voter than the offer that would be obtained by substituting \( E[\lambda] \) for \( \lambda \) in the solution to the complete-information case.

Part (v) implies that the appearance of decay on the equilibrium path is not a permanent phenomenon. Once the absorption point of \((\pi, 0)\) is reached the interests of the players are sufficiently aligned that the Proposer simply offers his ideal point each period and it is accepted by the Voter, even when the Voter knows the precise level of decay and the Proposer doesn’t.